ANALOGY BETWEEN THE PROCESSES OF MOMENTUM TRANSFER AND HEAT TRANSFER IN TURBULENT FLOW OF A LIQUID IN A TUBE AND IN A GRAVITATIONAL FILM

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On the basis of analysis and comparison of theoretical and experimental data on hydrodynamics and heat transfer, it is shown that in a number of cases there is an analogy between these processes in liquid flow in a tube and in a gravitational film.

As is known, in tubulent liquid flow in tubes, when the main resistance to momentum transfer and heat transfer is concentrated in the region near the wall, the use of an equivalent tube diameter as a determining dimension makes it possible in many cases to describe by similar equations the laws governing friction and heat transfer in tubes with different cross-sectional shapes. In the case of laminar flow, when the resistance to momentum transfer and heat transfer is of approximately the same order throughout the thickness of the boundary layer, the use of an equivalent diameter does not, as a rule, lead to the same friction and heat-transfer equations for tubes with different cross-sectional shapes.

The flow of a liquid in a film may also be regarded as flow in a tube having an appropriate equivalent diameter, which is determined from the equation

$$d_{eq} = \frac{4F}{U} = 4\delta_{fi}\left(1 \pm \frac{\delta_{fi}}{2R}\right). \tag{1}$$

The plus sign is taken for flow along the outer surface of a vertical tube, while the minus sign is taken for flow along the inner surface. In the case of a plane surface, R = ∞ and $d_{eq} = 4\delta_{fi}$.

In the case of the flow of a gravitational film of liquid, as in the case of liquid flow in a tube, in the initial segment there is stabilization of the hydrodynamic and heat-transfer processes. With regard to stabilization of hydrodynamic processes in a turbulent film, we can distinguish two types: stabilization of the flow and stabilization of the hydrodynamic structure of the film. Depending on the spraying density and the velocity of efflux from the distributing device, the flow velocity of the film in the initial segment may increase or decrease until a stabilized velocity, at which the gravitational forces are balanced by the frictional forces, is established. The length of the stabilization segment of the flow depends essentially on the value of $\varepsilon_{eff} = w_{eff}/\overline{w}$ and Re [1-3].

Stabilization of the flow and, correspondingly, of the average film thickness does not necessarily mean that the hydrodynamic structure of the turbulent film has been stabilized. Usually, after stabilization of the flow and the average film thickness, the thickness of the continuous layer of the film near the wall will decrease further, and the thickness of the wave layer will increase. These parameters of a turbulent film of water, according to [4], are stabilized, on the average, at a distance of 3-4 m from the distributing device. Reconstruction of the hydrodynamic process in the initial segment of the film affects local heat transfer. According to [4], the heat-transfer coefficient varies until there is stabilization of the continuous layer and the wave layer, i.e., at a distance of 3-4 m. According to the data of [2], as well as investigations conducted by the author and his coworkers, stabilization of the heat-transfer coefficient comes much earlier and coincides approximately with the stabilization of flow. Consequently, stabilization of heat transfer, like stabilization of flow, depends essentially on the values of ε_{eff} and Re.

Thus, in the initial segment of a tube and a film the development of hydrodynamic and heat-transfer processes may be completely different, and the analogy between these processes in a tube and in a film can exist only in individual special cases.

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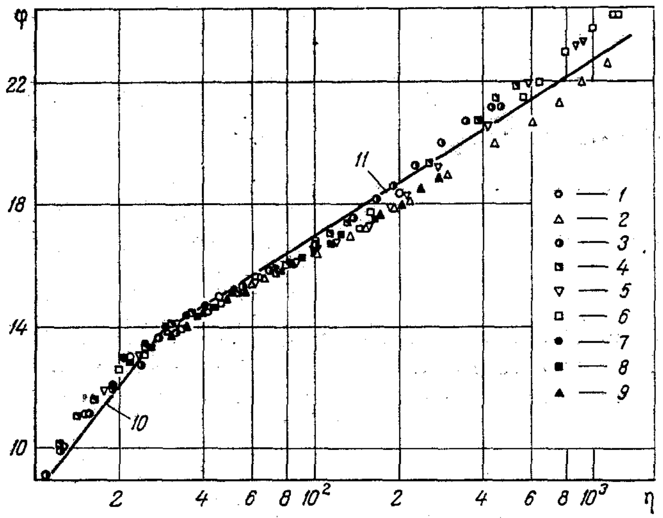

Fig. 1. Universal profile of velocities in a tube and in a film: 1) Re = 24,000; 2) 43,400, tube [5]; 3) 15,180; 4) 20,530; 5) 35,270; 6) 50,390, tube [6]; 7) 13,710; 8) 23,900; 9) 40,900, film, data of author and co-workers; 10, 11) according to formulas (2) and (3).

The situation is different in the case of stabilized flow in a tube and in a film. Figure 1 shows the profiles of velocities in fully developed turbulent flow in water (Re > 10^4) in a tube according to the data of [5, 6] and in a film according to measurements made by the author and his co-workers. The velocity in the film was measured by means of a velocity tube made from a medical needle [7]. The effect of the wall on the tube readings was taken into account in accordance with [8].

As can be seen from the figure, the distribution of velocities in the turbulent wall region in the tube and that in the film are analogous. For comparison, in Fig. 1 we also show the velocity profile obtained by the widely known Nikuradze–Kármán three-layer scheme:

$$\varphi = 5 \ln \eta - 3.05 \text{ for } 5 \leqslant \eta \leqslant 30, \tag{2}$$

$$\varphi = 2.5 \ln \eta + 5.5 \text{ for } \eta \geqslant 30. \tag{3}$$

For stabilized flow of a gravitational film of liquid along a vertical surface

$$\xi \frac{\rho \bar{w}^2}{8} = \tau_w = 0.25\, g\rho d_{eq}. \tag{4}$$

Consequently, the resistance coefficient ξ for gravitational film flow can be determined either by direct measurement of the tangential stress at the wall or by measurement of the average thickness of the film.

In the first case,

$$\xi = \frac{8\tau_w^3}{g^2\rho\Gamma^2}, \tag{5}$$

while, in the second case,

$$\xi = \frac{g\rho^2 d_{eq}^3}{8\Gamma^2}. \tag{6}$$

The author and his co-workers conducted investigations on ξ by the first method for turbulent and pseudolaminar flow of a film of water on the outer surface of a smooth vertical tube [7] and by the second method for the flow of a turbulent film of water on the inner surface of a smooth vertical tube [7] and for laminar flow of a film of transformer oil on the outer surfaces of a smooth vertical tube. The thickness of the oil film was measured by means of a needle and a coordinate mechanism.

The results of the investigations, together with the data of [9], are shown in Fig. 2.

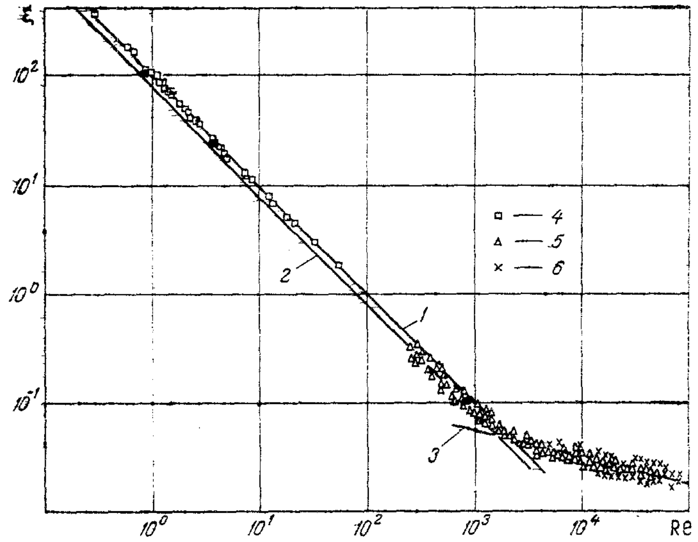

Fig. 2. Frictional resistance in a tube and in a film: 1, 2, 3) according to formulas (8), (9), and (7), respectively; 4) film of transformer oil; 5) film of water, data of the author and his co-workers; 6) film of water [9].

As is known, for calculation of the frictional resistance in smooth tubes with $3 \cdot 10^3 <$ Re $< 10^5$, the equation most widely used is the Blasius equation,

$$\xi = 0.316\,\mathrm{Re}^{-0.25}, \tag{7}$$

which, according to Fig. 2, can be used for calculating the frictional resistance for turbulent flow in a gravitational film of liquid as well.

Thus, the results of the measurement of the frictional resistance and the results of the measurement of the velocity field clearly show that there are similar laws governing the transfer of momentum in the wall region of the liquid in the case of turbulent liquid flow in a tube and in a film.

In Fig. 2, line 1 corresponds to the law governing resistance in laminar flow of a liquid in a plane or annular slit [10] and the law governing resistance in laminar flow of a film which is derived from Nusselt's theory [11]:

$$\xi = 96\,\mathrm{Re}^{-1}. \tag{8}$$

Consequently, in the case of laminar flow the analogy of momentum transfer is found in a film and in a slit. This fact is quite natural, since the flow in a film may be regarded as half of the flow in a slit. In this case,

$$d_{\mathrm{eq.sl.}} = 2\delta_{\mathrm{sl}} = 4\delta_{\mathrm{fi}} = d_{\mathrm{eq.fi}}$$

It can also be seen from Fig. 2 that for pseudolaminar flow of a film the experimental data correspond more closely to the law of resistance derived from Kapitsa's theory of wave flow in a film [12]:

$$\xi = 77\mathrm{Re}^{-1}. \tag{9}$$

Many experimental data on resistance in the flow of a liquid film which also correspond to Eqs. (7), (8), and (9) are given in [13].

The existence of an analogy of momentum transfer in a tube and in a film gives reason to suppose that there must also exist an analogy of heat transfer. Figure 3 shows data on stabilized heat transfer in the turbulent flow of a film of water along the surface of a smooth vertical tube, obtained by the author and his co-workers on an experimental apparatus which is described in [14]. It also shows analogous data found in [2]. In the calculation of the Nusselt numbers Nu the equivalent diameter of the film was determined from the equation

$$d_{\mathrm{eq.}} = 0.54\left(\frac{\nu^2}{g}\right)^{1/3}\mathrm{Re}^{7/12}, \tag{10}$$

which is obtained from a comparison of (4) and (7).

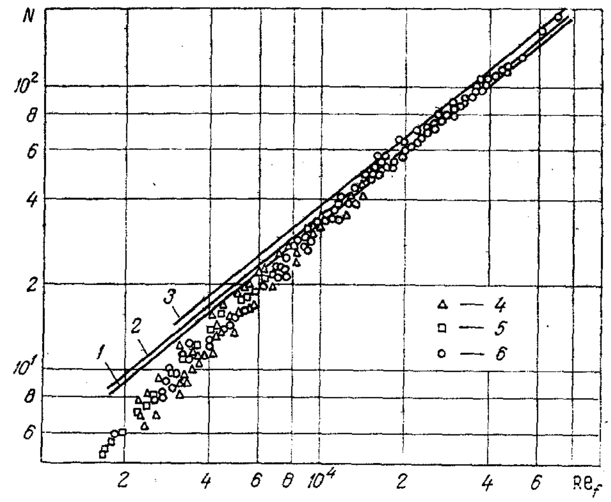

Fig. 3. Stabilized heat transfer in a tube and in a film: 1, 2, 3) according to formulas (11), (12), and (13), respectively; 4) heat transfer in a tube [15]; 5) heat transfer in a film [2]; 6) heat transfer in a film, data of the author and his co-workers. $N = Nu_f Pr_f^{-0.43}(Pr_f/Pr_w)^{-0.25}$.

Figure 3 also shows experimental data [15] on average heat transfer in long tubes ($L/d > 50$) in a transitional flow regime and the results of heat-transfer calculations by the well-known formulas of Mikheev, Sukomel, et al. and Petukhov–Kirillov, which are used for the calculation of heat transfer in smooth tubes for fully developed turbulent flow ($Re > 10^4$) [10]:

$$Nu_f = 0.021\, Re_f^{0.8} Pr_f^{0.43} \left(\frac{Pr_f}{Pr_w}\right)^{0.25}, \tag{11}$$

$$Nu_f = 0.022\, Re_f^{0.8} Pr_f^{0.43}, \tag{12}$$

$$Nu_f = \frac{\xi Pe_f}{8\left[1.07 + 12.7\sqrt{\frac{\xi}{8}}\,(Pr_f^{2/3} - 1)\right]}\left(\frac{\mu_f}{\mu_w}\right)^{0.11}. \tag{13}$$

As can be seen from Fig. 3, for $Re > 2500$ there is an analogy of heat transfer in the tube and in the film.

Formulas (11)-(13) describe heat transfer in smooth straight tubes with fully developed turbulent flow, i.e., with $Re > 10^4$. This is also found in the flow of a gravitational film along a smooth vertical surface. Consequently, the data given in Fig. 3 clearly show that fully developed turbulent flow in a film, as in a tube, takes place only when $Re > 10^4$, while when $Re < 10^4$ [approximately up to $Re \simeq (1.2)\cdot 10^3$], the flow regime is transitional.

The methods of theoretical calculation of heat transfer given in [14, 16, 17] yield the following Nusselt numbers for stabilized heat transfer in laminar flow in a film:

$$Nu \simeq 8.2 \text{ for } q_w = \text{const},$$
$$Nu \simeq 7.5 \text{ for } T_w = \text{const}.$$

According to [18], these are the values taken on by Nu in the region of stabilized heat transfer when there is laminar flow of a liquid in slits with heating on both sides. Consequently, in laminar flow an analogy of heat transfer, as well as of momentum transfer, is found in the film and in the slit.

According to the theoretical investigations of [17, 19], for turbulent flow of a liquid-metal film along a smooth vertical surface, in the absence of thermal contact resistance at the wall, the heat transfer can be determined from the following equations:

$$Nu = 8 + 0.036\, Pe^{0.8} \text{ for } T_w = \text{const}, \tag{14}$$

$$Nu = 9 + 0.04\, Pe^{0.8} \text{ for } q_w = \text{const}. \tag{15}$$

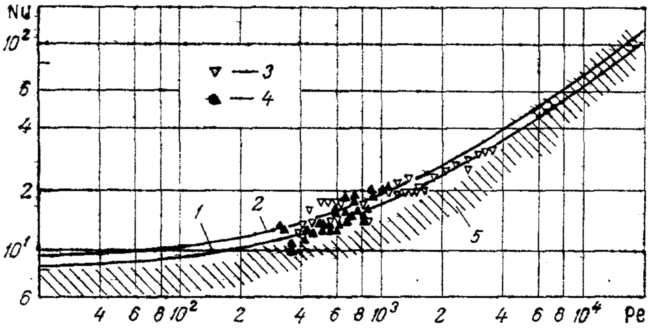

Fig. 4

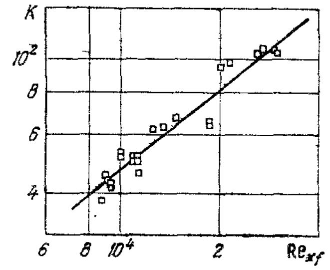

Fig. 5

Fig. 4. Heat transfer of liquid metals in tubes and in a film: 1, 2) by formulas (14) and (15); 3, 4) heat transfer in an annular channel with two-sided heating, from the data of [20] and [21], respectively; 5) region covered by the experimental data on heat transfer in a circular tube, according to [22, 23].

Fig. 5. Heat transfer in the thermal-stabilization segment for stabilized flow of a turbulent film. The solid line is plotted on the basis of formula (16). $K = Nu_{xf} Pr_f^{-0.43} (Pr_f/Pr_w)^{-0.25}$.

In Fig. 4 we compare these equations with experimental data on heat transfer in the flow of pure liquid metals in circular tubes and in slits with two-sided heating. As can be seen from this figure, for moderate values of Pe an analogy of heat transfer is found in metal flow in a film and in a slit, while for large values of Pe it is found in a tube as well. This is quite regular, since for moderate values of Pe (and Re), owing to the high thermal conductivity of liquid metals, the temperature profile is still close to a laminar profile, and an analogy exists only between heat transfer in a film and in a slit (as in the laminar flow of ordinary liquids).

We can give one more example of the analogy of heat transfer.

When before the heat-transfer segment there is a hydrodynamic-stabilization segment and there is a flow of a turbulent film with stabilized velocity onto the heat-transfer surface, we may expect the process of development of the thermal boundary layer to be qualitatively the same as when there is external flow of an unbounded stream past the surface and there is an initial isothermal segment. In this case, according to [10], the following equation may be used for the calculation of the local heat-transfer coefficients:

$$Nu_{xf} = 0.0296\, Re_{xf}^{0.8}\, Pr_f^{0.43} \left(\frac{Pr_f}{Pr_w}\right)^{0.25}. \qquad (16)$$

Here we take the coordinate x, measured from the beginning of the heat-transfer segment, as the determining dimension and we take the velocity on the boundary of the hydrodynamic boundary layer as the determining velocity.

The experimental data, processed in this way, for heat transfer in the thermal-stabilization segment for stabilized flow of a turbulent film of water are shown in Fig. 5. These data were obtained on an experimental apparatus which is described in [24].

As can be seen from Fig. 5, the experimental data are satisfactorily described by formula (16). Consequently, the process of development of the thermal boundary layer is analogous both for flow of a stabilized film past the heat-transfer surface and for flow of an unbounded stream past this surface, although in the first case the hydrodynamic boundary layer is already fully developed, while in the second case it is in the process of development.

As can be seen from Fig. 3, the best agreement with the experimental data for stabilized heat transfer of a film with $Re > 10^4$ is given by formula (12). Then, obtaining expressions for α from Eqs. (12) and (16) and setting the two equal to each other, we obtain an expression for determining the length of the thermal-stabilization segment when a turbulent film with stabilized velocity flows onto the heat-transfer surface:

$$x_{th.st} = 6.5\, d_{eq.}, \qquad (17)$$

or

$$x_{th.st} = 3.5\left(\frac{\nu^2}{g}\right)^{1/3}Re^{7/12}. \tag{18}$$

In deriving these equations and processing the experimental data shown in Fig. 5, we assumed (on the basis of measurements of the velocity field) that the velocity in the film for $y = \delta$ was $w_\delta = 1.1\bar{w}$, when $Re > 10^4$.

Thus, the above results show that in a number of cases there is analogy between the processes of momentum transfer and heat tranfer in the flow of a liquid in a tube and in a film. This fact is of both theoretical and practical importance, since, on the one hand, it reveals common rules governing the transfer of momentum and heat and, on the other hand, it makes it possible to use the same formulas for the practical calculation of these processes.

In conclusion, it should be noted that the above findings do not mean that we can speak of a universal analogy between the processes of hydrodynamics and heat transfer in the turbulent flow of a liquid in a tube and in a film, even if these processes are stabilized. For example, the investigations in [25-27] show that the rules governing the flow of a film of water change when surface-active substances, which reduce, primarily, the surface tension of the water, are added to it. However, the addition of surface-active substances to water flowing in a continuous stream in a tube usually does not change the rules governing the flow, provided that these substances do not produce a Thoms effect. An analysis of the data given in [28] shows that different rules will also apply when there is flow of a liquid in a continuous stream in corrugated tubes and when there is flow of a film of liquid over a corrugated surface.

NOTATION

F, cross-sectional area of the film; U, wetted perimeter; δ_{fi}, average thickness of film; R, radius of tube; y, distance from wall; d_{eq}, equivalent diameter of film or tube; w_{eff}, velocity of efflux of film from distributing device; $\bar{w}$, average velocity with stabilized flow of liquid in film or in tube; w, local velocity of film; Γ, spraying density; $\varphi = w/v^*$, dimensionless velocity; $\eta = yv^*/\nu$, dimensionless distance from wall; τ_w, tangential stress at wall; $v^* = \sqrt{\tau_w/\rho}$, dynamic velocity; ρ, density; ν, kinematic viscosity; μ, dynamic viscosity; g, acceleration of gravity; λ, thermal conductivity; q_w, specific heat flux at wall; T_w, wall temperature; $Nu = \alpha d_{eq}/\lambda$; $Nu_x = \alpha x/\lambda$; $Re = \bar{w}d_{eq}/\nu = 4\Gamma/\mu$; $Re_x = wx/\nu$; $Pe = Re \cdot Pr$. Indices: w, parameters of liquid at wall temperature; f, parameters of liquid at mean mass temperature.

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HYDRODYNAMICS OF A WATER–AIR FLOW IN A VERTICAL ANNULAR CHANNEL

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Measurements on the effects of flow rate and geometrical dimensions on the hydraulic-resistance coefficient of a vertical annular channel containing a rising water–air flow are reported.

Annular channels of the tube-in-tube type are widely used in heat exchangers and the like; the heat transfer is accelerated if a gas–water mixture is used [1]. There are many papers on the hydrodynamics of such flows in pipes and channels, but similar studies for annular channels are few and rather conflicting [2, 3]. We have therefore made measurements on the hydrodynamics of water–air flows in such channels.

The working section was a vertical lucite tube having an inner diameter d_2 = 28 mm and length 960 mm; the internal coaxial part was a polished nonferrous tube of diameter d_1 = 9, 12, 14, 16, and 24 mm or a lucite rod of diameter 20 mm. The outer tube has a length of 480 mm in the experiments with a rod of diameter 24 mm.

The centering was provided by a supporting disk and the top of the chamber. The centering device set up in the latter had interchangeable cones, which allowed us to direct the films of liquid into cyclones separately from the inner and outer surfaces of the annulus. The liquid was deposited on the walls by means of a cylindrical bubble chamber of height 70 mm and inner diameter 90 mm, which had an air-distributing grid at the bottom containing 80 holes of diameter 3 mm.

During the preliminary experiments, an equivalent diameter d_e = 8 mm was used to examine the effects of various designs of bubble chamber on the resistance of the working section. Although there was some variation in the distribution of the liquid over the surfaces of the annulus, the resistance was almost independent of the size and design of the bubble chamber.

The air was brought into the distributing grid by three DV-2 fans working in series. The air flow rate was measured with a Prandtl tube, an MMN-240 gauge, and a U-tube manometer, which indicated the static pressure.

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